An Optimal Method for Determining Tenant Mix (Including Location) in Shopping Centers

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October 31, 2010
Abstract: With assumptions like maximally productive lease structure, equilibrium space allocation, a just saturated retail market, and zero vacancies, store location would be the remaining variable in obtaining “ultimate tenant mix.” Here we produce an optimization program to maximize tenant location within shopping malls taking into consideration the two complementary affects present in malls: bid rent theory and revised central place theory.
1) Introduction

Some research has been done to explore the determinants of store/tenant location within shopping centers, yet not a lot is known specifically about where types and sizes of stores should best be located. There is a vague idea in the literature that some “ultimate tenant mix” (including tenants’ locations) may be discoverable, which would prove very useful to shopping center management in deciding what tenants should be included in a mall and in which locations.¹ No such magic formula exists, however, and research continues on the basis of adding to the knowledge of optimal store location and mix.² Some recent research has made some inroads towards solving the problem of ultimate locations for stores.

Research on location was made possible after basic knowledge was found on mechanisms affecting stores in malls. The academic findings generally parallel professional knowledge about the nature of store attributes and location.³ These findings were learned mostly during the 1990s, all under the rubric of “internalizing externalities.” It is the inter-store externalities that determine allocation of space, agency relationships, rent, location, and other characteristics of stores in shopping malls. What started with research on how retail leases differed with other commercial leases, for instance the price discrimination that takes place, led to the general determination of tenant rents, how space in shopping centers allocated space, determination of agency issues, and eventually to matters regarding tenant location.

¹ See, e.g., Marlow (1992) and Stambaugh (1978).

² A seemingly more subjective method has recently been put forth by Des Rosiers, et al. (2009), which proposes that certain notions are better than others, for instance less concentration of store types rather than more, but seems not to give hard and fast rules on tenant mix.

³ For instance, Bean, et al. (1988), at 2 report that Homart Development Company (a shopping center owner and developer) normally negotiates low lease rental rates with their anchor stores and that their “profits are made primarily from the rent paid by the non-anchor tenants.”
Generally, the following has been discovered (see, generally, Carter (2009)). 1) price discrimination takes place in shopping center leasing (Benjamin, et al. (1992)); 2) tenants (stores) vary by capability of generating customer traffic, and this capability ranges by store type; rent subsidies go to those that produce these externalities while rent premiums are paid by those that “free ride” on them; per foot subsidies and premiums have been measured (Pashigan & Gould (1998)); 3) other factors held constant, larger, newer, centers with national anchors, centers with higher traffic counts, those with less vacant space (Sirmans & Guidry (1993)) and those with greater market area purchasing power (Gatzlaff, et al. (1994)) charge more rent; 4) significant vacancy increases, especially among anchor tenants, cause rents to fall substantially after a while (Gatzlaff, et al. (1994)); 5) space is allocated to tenants (stores) to the point where net marginal revenue is equal to the marginal cost of space, less an externality term, all adjusted by each store’s elasticity of demand (Brueckner (1993)); 6) “common agency” tells us that tenants’ (stores’) leases should be a combination of fixed, base rent and percentage rent and include a lease cancellation provision (Miceli and Sirmans (1995)); and 7) fixed rent varies inversely to sales externalities generated by stores, and percentage rent varies positively with fixed rent (sales externalities) (Wheaton (2000)); 8) highest mall traffic takes place at the mall center and tapers off with distance from the center (Carter and Vandell (2005)).

On this basis the following has been discovered about tenant (store) location (See, generally, Carter (2009)). 1) Tenant (store) location varies, size per tenant (store) increases, rent paid per square foot for a tenant (store) decreases, and revenue earned per square foot of a tenant (store) increases, in accordance with distance from the mall’s center (Carter and Vandell (2005)); 2) tenant (store) types will line up according to their bid rent curves, those able to bid highest at

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4 Theory on this economic process can be found in the earlier works of Ghosh (1986) and West, et al. (1985).
each location with distance from the center (Carter and Vandell (2005)); 3) stores of the same type that are generally thought of as promoting comparative shopping will generally disperse as opposed to cluster in malls (Carter and Haloupek (2002)); 4) concentration of tenant (store) types, whether those promoting comparison shopping or not, tends to lower rents (and lower revenues) (Des Rosiers, et al. (2009); Eppli and Shilling (1995)).

This last finding merits some elucidation in the context of this paper. It’s fairly simple to see that too many of one store type (say men’s shoes) will have a negative effect on mall economics since it will create a too-competitive environment. Less numbers of this store type will provide a comparative, not a too-competitive, environment. The location research just mentioned says some dispersion rather than clustering of same-type tenants (stores) works best. So number and separation distance is one part of the location problem that needs to be estimated, whose solution would add to our knowledge of shopping centers. Finding the distance between stores of the same type would assist in determining the number of each store type, since there is a limited amount of space to the mall and just so many stores of a single type could fit into that space.

Use of this information whose parameters can be set out fairly exactly would lead to a reasonable answer to the question of the “ultimate tenant mix” (including locations) for a standard regional shopping center. The question then becomes how to best use these rules as inputs to form an optimal result (see Section 3). Before that it would be judicious to first discuss in some detail other characteristics of store location in malls.

5 Theory on this economic process can be found in the earlier works of McLafferty and Ghosh (1986) and Ghosh (1990). The concept is called revised central place theory and says same type non-anchor stores in a mall will disperse to where the marginal revenue benefit due to reduced rates of multipurpose shopping (and increased rates of single-purpose shopping) equal marginal revenue due to a smaller market.
2) A Closer Look at Tenant Location in Malls

We do not understand the problem as at all subjective, where discussion of “more” or “less” of some characteristic is better or worse for overall revenues or profits for stores or for overall revenues or profits for mall owners. The assumption has always been implied that all of the matters at work in shopping centers create a Pareto optimal equilibrium. As such any other location than that defined by the equilibrium solution would mean less revenue (profits) for some store (tenant) in the mall.

Looking at the Problem in Terms of Bulk Store Types

A recent article by Des Rosiers, et. al (2009) uses the Herfindahl index to measure the intra-category retail concentration of 31 categories of stores (tenants) located in 11 Canadian shopping centers, 6 centers in Montreal and 5 in Quebec. Not surprisingly they find that intra-category retail concentration affects base rent negatively “as a result of increased bargaining power enjoyed by dominant tenants.” But this affect ranged from low to high among the categories of non-anchor stores (tenants). Their finding that base rents follows percentage rents, ceteris paribus, confirmed a finding by Pashigan and Gould (1998) and Chun (1996). Overall, then, intra-category retail concentration lowered rents. Another finding, new to the literature, was that higher order stores (tenants) paid higher base rent than lower order stores (tenants), ceteris paribus.

The Heifendahl index measures the concentration of the production in an industry in terms of a retail unit’s gross leaseable area (GLA), measured from 0 (absence of a store in a retail category) to 1 (all retail activity put out by a single tenant). Each retail category is

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6 Existence of a Nash-Courtney equilibrium should follow (Yu (2002)).

7 Previous working papers along the same lines are Des Rosiers, et al. (2004) and Shun-Te Yuo, et al. (2003).

8 In the literature “high” order stores are those selling usually pricier goods for which customers compare shop; “low” order stores are those selling less expensive goods, those not requiring comparison shopping.
deemed a market for which store(s) (tenant(s)) hold a market share, and is more commonly used in economics as an measure of competition among firms in a market. The resulting index is proportional to the average market share, weighted by market share, ranging from 0 to 1.0, that is, ranges from a large number of small firms to a single monopolistic producer. The stores (tenants) here obviously represent all competitors in the “market” and what is understood as defining a “market” is of great importance.

In terms of the overview of shopping center knowledge given above, high concentrations of store categories resulting in lower base rents would represent too much GLA for the store type. Separation or dispersion of same-type stores would mean that at some point there would be a limit to the number of stores (or GLA) of that store type in a mall. At some point the amount of GLA of a store type would be suboptimal.

In working papers by Eppli and Shilling (1993 and 1995) the authors divide the stores into anchor stores and non-anchor (mall) stores. Attempts are then made to separately measure the customer drawing power or revenue produced by each. The second paper can be understood as consisting of the same sort of theoretical and empirical treatment as the first. Attempts are here made to distinguish between revenue (sales) generation made by anchor stores, called retail mass agglomeration, and revenue (sales) made by non-anchor stores, called retail merchandise type attraction. The authors conclude that more anchor store GLA and more non-anchor store outlets in shopping centers both result in higher revenue (sales) for the center. These results may seem to be explainable by the simple finding made that larger regional and super-regional shopping centers do better than smaller centers, ceteris paribus (Guidry and Sirmans (1993)). For that reason we assume they remained working papers, not subjected to review by peer-reviewed journals. But they do pose an interesting alternative to the Des Rosiers, et al. (2009)
model with regard to handling the issue of markets.

Significant parts of these two papers consist of the empirical attempt to estimate the market expansion potential of store (tenant) categories in a market area extending beyond the shopping centers. Here it is implicit that one category of stores (tenants) in the center do not make up the market. Consequently, much of the analysis using the Herfindahl Index by Des Rosiers, et al. (2009) would not be possible because it assumes market saturation (but not over-or under-saturation) for all store (tenant) categories.

**The Way to Look at It**

Here we will make the assumption that the market areas for the regional or super-regional shopping centers are saturated, that is, there is little if any room for growth (or size reduction) for stores of any type, total gross leaseable area (GLA) being in-line with population income and spending. Another assumption is that generally Pareto optimal solutions can be obtained by reviewing characteristics of store (tenant) types, sizes, and locations, that is, shopping center operations over many years have realized these results.

Optimum store mix may be a somewhat illusory concept. Store locations and sizes will go through iterations over time, that have more to do with what management thinks are good results at the time as opposed to what they think may be the best results at the time. The perfect outcome may never be realized but best tendencies should be always recognizable. With this in mind we will use inputs from research on tenant (store) location to locate “optimum tenant mix.”

**3) Optimum Tenant Mix**

**A) A Method for Determining Optimum Store Location**

One way to determine the “best” location of stores in shopping centers would be to treat
the matter as an optimization problem. In fact this method has already been done in the
management science literature, where the authors seem to be much more interested in the
algorithm than with the inputs and results. In “Selecting Tenants in a Shopping Mall” (Bean, et
al. (1988)) the authors create a nonlinear integer program that solves for the optimum of tenant
mix and locations, solving for the number of tenant types (20 store types) with those types’ sizes
(3 size classes) and 3 location classes (side aisle leading to a parking lot, main aisle between
anchors, and another class). An objective function optimizes present worth of the mall, which is
made up of the present values of stores in the mall. The exercise is premised on the idea that
rents of the mall tenants (both base rent and percentage rent) are the primary factor in the value
of a regional or super-regional mall.9

This exercise seems straightforward, except, of course, for the mathematics. But after
some reflection the reader should question what the inputs of this model really consist of. The
authors make no assumptions as to why or where certain tenants (stores) should be located and
what sizes these certain tenants should be at any location. These are given them by Homart’s
Market Research Group. On pages 4 and 5 the authors say:

“… Homart’s Market Research Group can estimate a store’s sales over the study horizon
given its type, size, location class, and the number of stores of its type in the mall. These
revenue figures, together with rental rates depending on the store’s characteristics,
determine the rental income Homart will receive. Then a present worth coefficient, $PW_{ijkl}$,
is calculated to be the total contribution to present worth of a store of type $i$, location
class $j$, and size $k$ if it is one of $l$ stores of type $i$ in the mall.

The why’s and wherefore’s of the inputs are evidently what the Market Research Group found as
the best historical rents received for each tenant (store), taking into consideration type, size,
location class, and number of stores in the malls. The malls’ sizes and shapes are not considered,
nor are consumers’ characteristics or market areas, nor distances from the malls’ center or store

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type dispersion. Consequently, their results are not at all based on theory. And their variables may have been important or not important and the ways they were employed may have been well or poorly done.

The authors confess to improving on earlier approaches to solving the tenant mix problem, and gained recognition from Homart Development Company that their results did indeed do a better job than the company had accomplished in the past. What this amounts to is telling Homart where to put stores of various types and sizes in a mall based on the past sales of different types, sizes, and general locations of its stores. Not much is said about what is going on regarding the effects of stores on each other. The authors do state that their data shows an optimum number of stores of each type exist. They base this on the fact that as the number of stores of each type grows, returns for each store initially grows, but at some point marginal gains decrease and then grow negative as stores begin to compete.

These authors use twenty (20) different store types, the number of stores (tenants) of each type, with three (3) different general mall locations, and three (3) sizes (within the store type’s range) of each store (tenant) type. The number of each store (tenant) types, their locations, and their sizes within a hypothetical mall that bring the highest value to the developer is their solution. This they compare to the mix and locations in actual malls and rents received. In essence the paper is an exercise in placing stores in those locations in a model mall where it was shown they produced best in the malls making up their data base (within their made up mall parameters).

3) Inputs from Tenant (Store) Location Research

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10 Nothing beyond the facts that some were new malls and some were relatively new malls was given.
The Data

For our study, tenant, lease, and location data on mall stores from regional and non-regional centers were supplied by two (2) sources that required confidentially. The database consists of 1,012 of stores (tenants) doing business during 1991 and 1992 from nine (9) centers located throughout the U. S.: Pacific 2, West North Central 1, East North Central 1, Southeast 2, Northeast 3. All centers are enclosed, of contemporary design, comparable in amenities and occupancy (nearly 100%).

Total shopping center sizes differed in size from 613,400 square feet to 1,004,400 square feet (819,650 average). The mall area of the shopping centers differed in sizes from 283,600 to 403,700 square feet (347,033 average). Six (6) centers were single level, two (2) had two (2) levels, and one had three (3) levels. For our purposes we used data from the six (6) single-level malls.\textsuperscript{11} Malls’ centers were discerned and stores’ distances from the center of each mall in which they were housed were measured and normalized so as to be comparable between malls. Explanations of the data gathering and processing are set out in Carter (1999), Carter and Vandell (2005), and Carter and Haloupek (2002).

Descriptive statistics for this sample are set out in Tables 1, 2, 3, and 4.\textsuperscript{12} Both sales and rents per square foot for stores generally and for store types tend to decrease at a decreasing rate with increased square footage. Average annual non-anchor sales for all nine malls was $361.44 per square foot. Average distances from the center of the mall (normalized) and average sales and rents per square foot are also set out. For individual malls, average non-anchor sales per

\textsuperscript{11} We feel the older, single level mall data give us a less complicated, more basic view of how shopping centers operate. Data from this period would not be in competition with "big box" retailers nor would they reflect more modern nuisances of today's various mall themes.

\textsuperscript{12} Distances to malls' centers (in feet) were normalized so as to be comparable across malls, as follows: square feet = (square feet/square feet of mall area) x 1000.
square foot ranged from $266 to $435. Rents varied similarly. Differences in revenues and rents varied according to income per capita in the shopping center’s market area, e.g., downtown San Francisco (highest) versus suburban Memphis (lowest). Vacancy did not vary a great deal, averaging 3.1%, and none of the vacancy levels was excessively high. Local tenants (businesses originating nearby) made up 37% of the database, while national chains (businesses having stores located in at least a couple of states and a common name) made up 63%.

Our data divide store types into eleven categories, including fast food (food-court). These eleven (11) categories have proven very useful, and we feel nothing would be gained by adding more. For our hypothetical we have chosen a standard “I” shape, small-sized, single level mall with two (2) separated anchor stores, much like the hypothetical regional shopping center set out in Dollars and Cents of Shopping Centers (Figure 1) and like the malls’ designs from our database (see Carter and Haloupek (2002), Figure 4). Since fast food goes in the middle, that leaves ten (10) store types to distribute in the mall (rather than twenty (20)). Since we feel location is a more important variable, the mall is divided into ten (10) sections instead of just three (3), based mostly on relative distance to the malls’ centers.

Spatially, fast food stores makes up about four (4) percent of store space, but tables, chairs, trash receptacles and so forth take up sizable space in the food court. Fast food is left in the center of the hypothetical shopping center while the remaining ninety six (96) percent is left as space for the other ten (10) store types. Taking the weighted average space using the descriptive statistics above, the rest of the stores take up the following percent of mall space shown below. Numbers of stores of the various store types are also shown for a model mall.
Table 1. Store Characteristics

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>square feet (SF)</td>
<td>$2,394.99</td>
<td>$2,233.14</td>
<td>120</td>
<td>27,000</td>
</tr>
<tr>
<td>sales ($/SF) (SALES)</td>
<td>$361.44</td>
<td>$217.12</td>
<td>$33</td>
<td>$1,632</td>
</tr>
<tr>
<td>total rent ($/SF) (TRNT)</td>
<td>$36.64</td>
<td>$25.71</td>
<td>$5.83</td>
<td>$277</td>
</tr>
</tbody>
</table>

Table 2. Store Characteristics (11 Store Types)

<table>
<thead>
<tr>
<th>Store type</th>
<th>Mean rent per SF</th>
<th>Rank</th>
<th>Mean size in square feet (standard deviation)</th>
<th>Mean Distance to Center (feet) (normalized)</th>
<th>Rank Distance (normalized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All stores</td>
<td>$37.75</td>
<td>2,417.06</td>
<td>295.3 (517.8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Jewelry</td>
<td>$63.58</td>
<td>1</td>
<td>1,239.57 (679.90)</td>
<td>281.6 (485.4)</td>
<td>3 (3)</td>
</tr>
<tr>
<td>(2) Cards and Gifts</td>
<td>$30.00</td>
<td>10</td>
<td>2,080.24 (1,219.88)</td>
<td>347.1 (596.7)</td>
<td>9 (9)</td>
</tr>
<tr>
<td>(3) Women’s Apparel</td>
<td>$30.00</td>
<td>11</td>
<td>3,906.92 (2,475.24)</td>
<td>312.7 (519.8)</td>
<td>5 (5)</td>
</tr>
<tr>
<td>(4) Fast Food</td>
<td>$59.05</td>
<td>2</td>
<td>874.69 (859.13)</td>
<td>70.5 (115.4)</td>
<td>1 (1)</td>
</tr>
<tr>
<td>(5) Family Apparel</td>
<td>$36.97</td>
<td>4</td>
<td>3,053.07 (1,861.71)</td>
<td>330.2 (547.9)</td>
<td>6 (6)</td>
</tr>
<tr>
<td>(6) Men’s Apparel</td>
<td>$32.97</td>
<td>6</td>
<td>2,384.16 (1,575.56)</td>
<td>336.0 (556.6)</td>
<td>7 (7)</td>
</tr>
<tr>
<td>(7) Leisure &amp; Entertainment</td>
<td>$33.85</td>
<td>5</td>
<td>2,651.53 (3,107.16)</td>
<td>405.1 (688.7)</td>
<td>11 (11)</td>
</tr>
<tr>
<td>(8) Home Furnishings</td>
<td>$31.40</td>
<td>7</td>
<td>2,666.49 (3,857.35)</td>
<td>326.7 (571.4)</td>
<td>8 (8)</td>
</tr>
<tr>
<td>(9) Men’s &amp; Boy’s Shoes</td>
<td>$30.43</td>
<td>9</td>
<td>2,397.19 (1,173.98)</td>
<td>396.6 (670.4)</td>
<td>10 (10)</td>
</tr>
<tr>
<td>(10) Women’s Shoes</td>
<td>$30.95</td>
<td>8</td>
<td>1,733.78 (993.78)</td>
<td>273.0 (453.7)</td>
<td>2 (2)</td>
</tr>
<tr>
<td>Variable/store type</td>
<td>Mean</td>
<td>Std Deviation</td>
<td>Min.</td>
<td>Max.</td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td>------------</td>
<td>---------------</td>
<td>------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>SF (Gifts)(6.5%)</td>
<td>2,088.25</td>
<td>1,172.81</td>
<td>665</td>
<td>5,475</td>
<td></td>
</tr>
<tr>
<td>SF (Women’s Shoes)(5%)</td>
<td>1,733.98</td>
<td>993.78</td>
<td>1,000</td>
<td>6,437</td>
<td></td>
</tr>
<tr>
<td>SF (Women’s apparel)(18%)</td>
<td>3,906.92</td>
<td>2,475.24</td>
<td>569</td>
<td>13,915</td>
<td></td>
</tr>
<tr>
<td>SF (Jewelry)(6.2%)</td>
<td>1,300.72</td>
<td>647.53</td>
<td>472</td>
<td>4,278</td>
<td></td>
</tr>
<tr>
<td>SALES ($/SF)(Gifts)</td>
<td>$298.82</td>
<td>$123.18</td>
<td>$136</td>
<td>$711</td>
<td></td>
</tr>
<tr>
<td>SALES (Women’s shoes)</td>
<td>$303.53</td>
<td>$116.99</td>
<td>$84</td>
<td>$566</td>
<td></td>
</tr>
<tr>
<td>SALES (Women’s apparel)</td>
<td>$258.18</td>
<td>$112.85</td>
<td>$90</td>
<td>$751</td>
<td></td>
</tr>
<tr>
<td>SALES (Jewelry)</td>
<td>$676.48</td>
<td>$276.02</td>
<td>$213</td>
<td>$1,433</td>
<td></td>
</tr>
<tr>
<td>TRNT ($/SF)(Gifts)</td>
<td>$30.95</td>
<td>$12.14</td>
<td>$16</td>
<td>$64</td>
<td></td>
</tr>
<tr>
<td>TRNT (Women’s Shoes)</td>
<td>$31.95</td>
<td>$9.60</td>
<td>$12</td>
<td>$51</td>
<td></td>
</tr>
<tr>
<td>TRNT (Women’s apparel)</td>
<td>$28.25</td>
<td>$16.83</td>
<td>$5.83</td>
<td>$77</td>
<td></td>
</tr>
<tr>
<td>TRNT (Jewelry)</td>
<td>$59.82</td>
<td>$27.11</td>
<td>$17.31</td>
<td>$127</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Characteristics by Selected Store Type
### Table 4. Percentage Store Types and Steepness of Bid-Rent Curves

<table>
<thead>
<tr>
<th>Store Type</th>
<th>Percent of Mall</th>
<th>No. of Stores if Same Size</th>
<th>Total Stores SF</th>
<th>Response Curve at Center</th>
<th>Degree Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) fast food:</td>
<td>3.98 percent</td>
<td>11</td>
<td>9,621.59</td>
<td>$64.37</td>
<td>2</td>
</tr>
<tr>
<td>2) women’s apparel</td>
<td>29.5 percent</td>
<td>18.5</td>
<td>71,331.19</td>
<td>$29.81</td>
<td>11</td>
</tr>
<tr>
<td>3) leisure &amp; entertainment</td>
<td>24.1 percent</td>
<td>22</td>
<td>58,333.66</td>
<td>$46.71</td>
<td>5</td>
</tr>
<tr>
<td>4) family apparel</td>
<td>8.8 percent</td>
<td>7</td>
<td>21,371.49</td>
<td>$37.58</td>
<td>6</td>
</tr>
<tr>
<td>5) home furnishings</td>
<td>6.8 percent</td>
<td>6.2</td>
<td>16,532.24</td>
<td>$36.50</td>
<td>9</td>
</tr>
<tr>
<td>6) men’s and boy’s shoes</td>
<td>6.6 percent</td>
<td>6.69</td>
<td>16,061.17</td>
<td>$36.50</td>
<td>10</td>
</tr>
<tr>
<td>7) men’s apparel</td>
<td>6.2 percent</td>
<td>6.3</td>
<td>15,020.21</td>
<td>$49.07</td>
<td>4</td>
</tr>
<tr>
<td>8) cards and gifts</td>
<td>5.6 percent</td>
<td>6.5</td>
<td>13,521.56</td>
<td>$37.59</td>
<td>8</td>
</tr>
<tr>
<td>9) jewelry</td>
<td>5.6 percent</td>
<td>6.20</td>
<td>7,685.33</td>
<td>$75.18</td>
<td>1</td>
</tr>
<tr>
<td>10) women’s shoes</td>
<td>3.6 percent</td>
<td>5</td>
<td>8,668.90</td>
<td>$40.99</td>
<td>7</td>
</tr>
<tr>
<td>11) specialty food</td>
<td>3.3 percent</td>
<td>6.20</td>
<td>7,889.56</td>
<td>$51.14</td>
<td>3</td>
</tr>
</tbody>
</table>

Total: 100%  
Total Doubled: 200 483,412

**Figure 1**
**Bid-rent Effect on Store Location**

Store type locations are figured using results found in Carter (1999) and Carter and Vandell (2005) on how store types line up in terms of the malls’ centers on the basis of a revised bid-rent theory and estimates. In that study the authors showed store types’ rents tapered off at different rates as they located further from the malls’ centers. Here we determine the relative losses of rent per square feet for the different store types, and figure where stores would range along the stretch from malls’ center to malls’ end, where the mall meets an anchor store. Standard deviations of store type rents are also considered in determining maximum and minimum limits for tenant types and rent per square foot in the optimization program set out below. A model “I” shaped, small regional shopping center houses 100 stores of the eleven (11) different store types, making up 241,706 square feet.\(^\text{13}\)

As shown above, jewelry stores have the steepest bid rent curve and, consequently, locate nearest the center of the mall, followed by fast food (food-court), specialty food, men’s apparel, and the rest. The least steep bid-rent curve was for women’s apparel, whose average size was the largest of the store types. Both expansions of the mall area from the mall center will be identical for the model “I” shaped shopping center, since store locations and store characteristics, store types and sizes, are meant to be identical on both sides.

**Revised Central Place Theory Effect on Store Location**

\(^{13}\) A limitation on data may include that only the distance of stores from the center are normalized and can be readily compared from one shopping center to another. Other data, store size, rents per square feet, and sales per square feet, are from all of the shopping centers and will not be readily comparable for several reasons, including size of the shopping center and income differences of the populace making up the trade areas. See, e.g., Guidry and Sirmans. It should be noted, however, that these limitations did not prevent very useful analysis, e.g., Pashigan and Gould (1998).

\(^{14}\) This was the result of many of the food-courts not being located in exactly the center of the malls from which the data was taken.
The second factor affecting store location is the dispersion of same-type, comparison shopping stores as demonstrated in Carter and Haloupek (2002). So, in addition to the bid rent factor there is the tendency for stores where customers comparison shop to form a cluster. The cluster isn’t one where stores of the same type all locate next to each other. Rather, they locate near enough to each other that the average shopper can make the most of the average shopping trip of three (3) store visits per trip. This means locations such that make for convenient trips for about two store visits for single purpose shopping and about four store visits for multi-purpose shopping. The results for stores of the same type is that they locate on both sides of the malls’ center and not usually next to each other.

In Carter and Haloupek (2002) the comparison shopping stores were women’s apparel, men’s apparel, men’s and boy’s shoes, and women’s shoes. To this list we add family apparel, cards and gifts, and jewelry stores, leaving fast food (food-court), home furnishings, and specialty food. Fast food (food-court), of course, makes up a class of its own, while home furnishings arguably sells “high-order” or shopping goods, not comparison goods. Though specialty food (e.g., Cinnabon, Aunt Anne’s Pretzels) is hard to categorize, it doesn’t seem to fit the category of comparison shopping goods used here.

4) The Quadratic Assignment Location Model

The problem most simply can be viewed as a quadratic assignment location problem with a linear objective function. The objective function maximizes the landlord’s/developer’s rents. Maximizing rents is largely a matter of maximizing mall stores’ (tenants’) rents, since anchor

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15 The main article on this theory taken from the literature was Ingene and Ghosh (1990), though other articles preceded that one (see, e.g., Eaton and Lipsey (1979) and De Palma, et al. (1985).

16 See,
stores receive the rent subsidies and the landlords’ (developers’) profits come entirely from the mall area.\textsuperscript{17}

Without fast food (food-court), the ten (10) stores’ space breaks down into the following square feet per store category and store numbers:

1) 71,331 SF women’s apparel (WA) (18.5 stores)  
2) 58,334 SF leisure and entertainment (LE) (22 stores)  
3) 21,371 SF family apparel (FA) (7 stores)  
4) 16,532 SF home furnishings (HF) (6.2 stores)  
5) 16,061 SF men’s & boy’s shoes (MS) (6.7 stores)  
6) 15,020.42 SF men’s apparel (MA) (6.3 stores)  
7) 13,522 SF cards & gifts (CG) (6.5 stores)  
8) 7,685 SF jewelry (J) (6.2 stores)  
9) 8,669 SF women’s shoes (WS) (5 stores)  
10) 7,890 SF specialty food (SF) (6.2 stores)

The store categories will line up, from closest to furthest from the food-court (furthest to closest to each of the anchors) in both directions of the mall. Their order in both directions follows steepness of their store types’ bid-rent curves as set out in Table 4. With two (2) anchor stores, one at each end of the mall, the linear mall area is about 720 feet long and the aisles are 30 feet wide. Stores are rectangular, about 130- 150 feet deep and about 30 feet wide. For instance, a large store making up 4,500 square feet might be 150’ x 30’, while a smaller store of 2,700 square feet might be 135’ by 20.’\textsuperscript{18} Total square feet of store space is about 250,000 square feet.

The model mall can most simply be viewed as four banks of stores, one on each side of the aisles, two aisles reaching out in opposite directions from the mall center (food-court). In this way the assumption is that the optimized bank of stores’ dimensions will be quadrupled, and these four banks of stores will make of the stores in the mall. The single bank of stores is as

\textsuperscript{17} See, e.g., Gould and Pashigan (2005).

\textsuperscript{18} Side aisles in actual malls help to balance out the differences between large, rectangular stores and smaller stores whose lengths are not so deep.
in the malls allowed.

The question of what rent is owed has to do with store-type rent spreads around their means. Rents for different store types were found to be normally distributed around their means.\textsuperscript{21} Normal distribution of rents was determined after the data was transformed so that rents among the several malls was comparable, much as distance measure data were normalized in Carter (1999) and Carter and Vandell (2005).\textsuperscript{22} Using the formula for the normal or Gaussian distribution to represent rents, a modest increase in height (median rents) and narrowing of width (lower standard deviation) of store-type rents would be what a mall owner (developer) would pursue, by locating stores of the same store type more in line with their optimal distances from the mall center.

A decrease of one tenth (10\%) of the variance in the rent distribution, by moving stores of a certain type closer together around their mean distance from the mall center, would result in an increase in mean rents of about five and four tenths percent (5.4 \%). Retaining comparison shopping stores’ dispersion as set out in Carter and Haloupek (2002) would be part of an overall distribution so as to maximum rents. Taking the normal or Gaussian function

\[
f(x) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

Mean rents will be increased to reflect this increase of 5.4\%. Rents are increased by 2.7\% to reflect an increase in rents on account dispersion of stores of the same type in the manner set out in Carter and Haloupek (2002), that is, more reflective of that pattern. The quadratic assignment programs’ distance and weight (connection) matrixes, respectively, are as set out in Appendix A.

\textsuperscript{21} The Kolmogorov-Smirnoff and Shapiro-Wilks tests were used.

\textsuperscript{22}
In the connection matrix, highest mean rent ($63.58/SF for jewelry) x 1.054 or $67.01/SF, and $67.01/SF x 1.027 = $68.82/SF. $67.01/SF represents rent for jewelry stores whose rent performance is raised 5.4% through better location, that is, the stores better approximate ideal bid rent locations. $68.82/SF represents jewelry store rent benefitting both from bid rent and revised central place theory (dispersion), here only one jewelry store. Each of the ten store types are represented in this fashion, e.g., for Men’s Apparel, $32.97/SF x 1.054 = $34.75/SF, and $34.75/SF x 1.027 = $35.69/SF.

What’s left is inputting different sized stores for each store type such that different sizes reflect the standard deviation or variance of store type sizes observed. The full LINDO problem and results are set out in Appendix B. The quadratic assignment location problem is represented as follows:

Maximize \( \sum_{i=1}^{n} RI_i S_i X_i \)

Subject to:

1. \( \sum_{i}^{m} \sum_{j}^{p} S_{i,j} X_{i,j} \geq LS_j \), (min. space allocated to a tenant type)
2. \( \sum_{i}^{m} \sum_{j}^{p} S_{i,j} X_{i,j} \leq US_j \), (max. space allocated to a tenant type)
3. \( \sum_{i}^{m} \sum_{j}^{p} S_{i,j} X_{i,j} \geq LL_j \), (min. space allocated to a tenant of a tenant type)
4. \( \sum_{i}^{m} \sum_{j}^{p} S_{i,j} X_{i,j} \leq UL_j \), (max. space allocated to a tenant of a tenant type)
5. \( \sum_{i}^{n} S_{i,j} X_{i,j} \leq GLA \ 250,000 \text{ SF} \)

where

RI = rental income per square feet for a specific tenant type
S = space for a specific tenant type
\( X_i = 1, 0 \)
LS = min. amount of space allocated to a specific tenant type
US = max. amount of space allocated to a specific tenant type
LL = min. amount of space available for tenant of a tenant type
UL = max. amount of space available for tenant of a tenant type
GLA = shopping center gross leaseable area (250,000 SF)

n = number of tenants (100), for i = 1, 2, ..., n
m = number of tenant types (10), j = 1, 2, ..., m
p = number of tenant type sizes (3), k = 1, 2, ..., p

5) Results and Conclusions

The objective function show annual rental income to the manager/developer to be $
. Multiplied by four (4) (the four (4) banks of stores making up the mall area) total rents for the
shopping center is $. Our optimization program takes into consideration all location
factors operating to affect rents. Since it is 1) assumed mall space is homogeneous (except
for build-out), 2) the retail market for all goods in shopping centers is assumed to be just
saturated (not over- or under-saturated), 3) leases are assumed maximally productive according
to theory, and 4) equilibriums affecting rent are assumed to exist, according to theory, then about
the only matters affecting rent levels have to do with location.

REFERENCES


The authors would like to especially thank Dr. Arthur N. Jensen for his help by sending us a copy of his dissertation and discussing it with us,
### Appendix A

**distance matrix**

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
```

### Appendix B

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|     | 0 | 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |
|-----|---|----|----|----|----|---|----|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0   | 10| 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |
| 0   | 10| 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |
| 0   | 10| 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |
| 0   | 10| 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |
| 0   | 10| 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |
| 0   | 10| 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |
| 0   | 10| 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |
| 0   | 10| 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |
| 0   | 10| 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |
| 0   | 10| 10 | 10 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 | 7 | 5 | 6 | 4 | 5 | 3 | 3 | 3 | 4 | 2 | 3 | 1 | 2 | 0 |

Appendix C

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