The 4% Rule: Does Real Estate Make a Difference?

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DRAFT – Prepared for the ARES Annual Meeting, April 2015

Current version: **March 31, 2015**

Abstract

This paper investigates to what extent, if any, real estate alters the findings of the 4% rule literature, which is based on an asset mix that does not include real estate. This is done by estimating the degree to which real estate holdings are likely to affect the ability of investors to withdraw funds according to the 4% rule. Specifically, we explore whether real estate holdings, in addition to a mix of stocks and bonds, decrease the probability of fund depletion and/or increase the amount investors are able to withdraw over long periods of time. Additionally, we provide an optimal range of real estate holdings as a portion of total assets that is likely to yield the most desired combination between withdrawal rate and fund depletion probability. A Monte Carlo simulation technique is employed that considers past performance of different asset classes, as well as the cross-sectional and serial correlation among the return series. Our real estate returns are the NFI-ODCE and the NAREIT equity REIT index quarterly return series. This paper intends to contribute to the existing literature on the 4% rule by (1) introducing real estate as an additional and independent asset class, and (2) using an improved methodology. The results of this work will have implications for endowments, pension funds, insurance companies as well as individual investors.

The 4% Rule: Does Real Estate Make a Difference?

I. Introduction

Much of investment research is focused on the optimal way in which to accumulate wealth. In actuality, however, the end goal of an investor is not only growing wealth, but the eventual spending of that accumulated wealth. This is most obvious in retirement planning, in which the objective of the accumulation phase (saving and investing) is to fund the decumulation phase (retirement spending).

Much of the financial planning industry is centered on advising retirees on how much they can spend during their decumulation phase. Often, this advice takes the form of a widely adopted rule-of-thumb known as the "4% Rule." In its simplest form, the 4% Rule states that a retiree should expect to withdraw 4% of their savings balance (i.e., wealth at retirement), adjusted for inflation, from their savings each year over their retirement. Various studies going back to at least Bengen (1994) have shown such a spending rule to have a relatively low risk of financial ruin in retirement (i.e. running out of money before the end of the investment horizon), although the evidence is certainly not universal (see, e.g., Pfau (2010)) . Scott, Sharpe, and Watson (2009) provide an excellent overview of the 4% Rule, including prior research and a discussion of its ubiquity in the financial planning industry.

Until recently, a discussion of retirement planning and spending rules for individuals could not have involved private market real estate. Real estate not only trades in an illiquid market that is relatively opaque and with high transaction costs, it is also characterized by assets that are large and typically indivisible.[1] Thus, while private market real estate has for many years been a widely accepted asset class for defined benefit (DB) pension plans, for all but high-net-worth individuals, real estate has not been a relevant option when planning personal savings and retirement plans. However, recent developments in the market are changing this. In response to the rapid growth of defined contribution (DC) retirement plans, and the lack of growth in the overall DB market, the real estate investment management industry has launched private real estate products designed to be suitable for inclusion in DC plans. In order to meet the needs of the DC plans, these relatively new real estate products are typically designed to

[1] While various partnership structures can be used to divide ownership interests in real estate assets amongst investors, these are generally not a practical option for individual investors who are not experts or do not have sufficient wealth to justify hiring expert advice.

provide daily liquidity (to facilitate capital inflows and outflows) and daily pricing. While no universally accepted structure for these DC real estate funds exists as of yet, liquidity is generally available through use of an allocation to cash, an allocation to REITs, and/or a credit line from the parent firm. Despite the use of quarterly appraisals for properties held by the funds, daily pricing of fund units can also be accomplished in several ways; one example involves reappraisal of any property upon any material event (such as signing a new lease, or losing a tenant) and/or staggering the appraisals of the properties across the quarter. Whatever the methodology behind these funds, many of the largest real estate investment managers have now launched private market real estate products for the DC market, meaning that private market real estate is now becoming a viable option for investment by individual savers and retirees.

In this paper we examine the ability of an allocation to real estate to help sustain spending during the decumulation phase of an investor's life-cycle. This research, therefore, brings together two separate strands of the prior literature, the literature on the 4% Rule and retirement spending rules in general, and the nascent literature on real estate in DC plans. Esrig, Kolasa, and Cerreta (2013) show that the addition of a 10% real estate allocation to a DC portfolio significantly reduced various risk measures while having no significant effect on average return. Drew, Walk, and West (2014) use a Monte Carlo approach to show that a real estate allocation can enhance the risk-return performance of a DC portfolio and can improve the probability of attaining a wealth-to-final salary goal at retirement. Both the Esrig, et al. and Drew et al. papers examine a mixed allocation to real estate, using both private and public market real estate, as that is standard amongst DC real estate products. Farrelly and Moss (2014) examine the effect of including a public market (i.e., REIT) component within a (mostly) private market real estate DC product and report that the public market component can increase return without diminishing the diversification benefits of private market real estate.

All of the prior research on real estate in DC plans, however, is concerned with the accumulation phase during which an investor builds wealth. In contrast, our research looks specifically at the decumulation phase and whether real estate can help sustain retirement spending plans. Harrison, Blake and Key (2013) note that "Real estate appears to be a very attractive asset to hold in a pension fund portfolio during both the accumulation stage of a DC scheme and – in due course – the decumulation stage" as well as stating that "arguably the role of real estate also extends beyond the glide-path into retirement." They do not, however, provide any empirical evidence on the topic. Our paper, therefore, represents the

first look at real estate in the decumulation phase portfolio, and its role in sustaining retirement spending plans.

Our results, based on a Monte Carlo simulation approach, show that portfolios optimized to minimize shortfall risk (i.e., the risk of spending not being sustainable) contain a large allocation to real estate. However, the form of the real estate, private or public, that is optimal depends on the situation. If only one form of real estate is used in the portfolio, then private market real estate tends to do slightly better at reducing shortfall risk than do REITs when the real estate allocation is constrained to be small (i.e. 5% or less). However, when the allocation to real estate is less constrained REITs clearly do a better job of helping to reduce shortfall risk in the portfolio. When both forms of real estate can be combined into an overall real estate allocation, REITs alone can be used to minimize shortfall risk when the real estate allocation is fairly large (20%); private real estate provides little or no additional help in sustaining a spending plan over an investment horizon in this case. However, if the overall allocation to real estate is constrained to be a relatively small part of the overall portfolio (5%), a common circumstance in practice, then a combination to REITs and private real estate is optimal. We also examine the case where an investor not only wishes to maintain a rate of spending, but also maintain the real value of the portfolio balance (i.e., wealth preservation). Results are similar in this case, but REITs are far more dominant. Private real estate does not seem to be as effective at sustaining spending while minimizing the risk of wealth falling below the inflation-adjusted initial level as it is at helping to sustain a spending level without regard to portfolio balance.

The remainder of this paper is structured as follows: In the next section we outline our methodology and the data used. In Section III we present the results of our Monte Carlo simulations, and the final section provides some avenues for future work.

II. Data and Methodology

We use a Monte Carlo simulation approach to examine the role of real estate in sustaining spending rules over a fixed investment horizon. In our simulations, funds held in an investment account are invested in three major asset classes; namely, stocks, bonds, and real estate. To be consistent with prior research on the 4% Rule (see Scott, Sharpe, and Watson (2009), for instance), our base case without real

estate is a portfolio of 60% stocks and 40% bonds. We treat any allocation to real estate as resulting in a pro rata reduction in the allocations to stocks and bonds; therefore, the portion of the portfolio not invested in real estate is always divided 60/40 between the other asset classes. We examine both private market real estate, public real estate, and a real estate allocation involving a combination of both.

We begin our simulations by assuming that an initial amount (X_t) is deposited into an investment account at the beginning of time period t. A predefined amount (Y), defined as a fixed percentage of X_0 (initial wealth), is withdrawn from the account before the remaining amount is invested. The funds then grows at an inflation-adjusted rate of return (R_t, see details below) that occurs during period t to equal X_{t+1} at the end of the period. Given that the expected return on the portfolio is calculated on an inflation adjusted basis, each withdrawal rate reflects a stream of cash flows that is constant in real terms. This procedure is repeated, each period corresponding to one year, until the investment horizon is reached. More formally:

$$X_{t+1} = (X_t - Y) * (1 + R_t)$$

The investment account balance is then recorded every five years over the full investment horizon.

While our initial motivation is to examine the role of real estate in the 4% Rule of retirement spending, different simulations are run based on a range of withdrawal rates to see if the results vary depending on the spending rate. Specifically, we examine spending rates of 3.5%, 4%, 4.5%, and 5%. We also examine a variety of investment horizons.

The rate of return for each time period is determined by our Monte Carlo simulation technique. The returns for stocks, bonds and real estate for time t are selected from historical annual inflation-adjusted return distributions that span the 1978 to 2013 time period. Specifically, the distribution for stocks returns is based on the value weighted historical returns of all CRSP firms incorporated in the US.[2] The distribution for bonds returns is based on the historical returns on the Barclay's US Aggregate bond index. The real estate distribution of returns is based on the historical returns of the NAREIT equity REIT index, for public

[2] Values are obtained from Dr. Kenneth R. French's Data Library: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

real estate, and NCREIF's ODCE index, for private real estate. Note that ODCE is an index of returns to open-end, direct-property real estate funds, which are similar in structure to the types of funds used in DC-eligible private real estate products. All inflation adjustments to the historical returns are calculated using the general consumer price index (CPI) during each year.

Returns on all asset classes are net of fees. We assume that, where possible, investors use passively managed funds mimicking the indices. We therefore deduct fees of 0.11% per year from bond returns, and 0.12% per year from equity and REIT returns based on the average expense ratio for passive bond and stock funds reported for 2013 according to the Investment Company Institute.[3] Passive investments are not possible for private market real estate as it is not possible to buy an index portfolio. We use the ODCE index returns net of fees, reflecting actual fee expenses charged to investors. For comparison purposes, the average difference between annual gross and net ODCE returns from 1978 to 2013 was 1.08%; in terms of management fees, private market real estate is a more expensive asset class than public market alternatives.[4] Table 1 provides summary statistics on the nominal, net of fees returns to the asset classes under consideration, as well inflation.

Note that we do not de-smooth private market real estate returns. There are two reasons for this. First, our use of annual returns, rather than quarterly, will mitigate, but not cure, any effects of appraisal smoothing. Second, and most importantly, in a DC context these returns represent the actual returns faced by investors. In other words, capital flows in and out of a private market real estate DC fund will take place at net asset value (NAV). Whether or not the reported NAV is an entirely accurate reflection of the true values of the underlying properties (due to appraisal smoothing or any other issue) is, from the investor's perspective, irrelevant. The investor puts in capital at NAV and gets NAV back when redeeming, and therefore returns based on reported NAV are the "true" or realized returns to investors in this context.

Much of the diversification benefit of investing in three asset classes is due to the fact that the returns from these asset classes are not perfectly correlated during each time period and from one period to the next. In order to capture the full diversification benefits of investing in stocks, bonds and real estate we

[3] See http://www.icifactbook.org/fb_ch5.html

[4] Of course, this comparison is only of fees on private real estate funds to fees charged by portfolio managers on public securities. A full comparison of the fee structures in private and public real estate would include the fact that many fees charged by private real estate funds include costs that would be included in the G&A expenses of REITs and therefore ultimately paid by investors, although not classified as a "management fee".

restrict the Monte Carlo simulation to withdraw values from the historical distributions given the historical inter-period and time-series correlation among the three asset classes during the 1978 to 2013 time period. This ensures that returns for each asset class during each time period are selected simultaneously with the appropriate correlation rather than independently in random. For each scenario of portfolio allocation we run the Monte Carlo simulation with 5,000 iterations.

Because the goal of the investment is dependent on the type of the investor, our analysis separately considers two types of investors. The first type of investor aims to minimize the probability that annual withdrawals would fully deplete the invested funds before the full investment horizon has passed a particular point in time, a condition referred to as shortfall or financial ruin. This type of investor, as envisioned in the original formulation of the 4% Rule, might be a retired individual that wishes to maintain a constant standard of living during retirement without "running out of money" at any point during his or her life time. In addition, we allow for a second type of investor with a slightly different goal. The objective of this second type of investor is to again withdraw a set amount, in real terms, from the account each year over the horizon, but in doing so maximize the probability that the ending balance in the account is at least as large as the initial wealth (in real terms). This second type of investor could be interpreted as a retiree who wishes not only to live off of their savings but also to leave a bequest. Alternatively, this investor could also be thought of as an endowment or foundation which wishes to provide fixed real spending power to its underlying organization while preserving the future value of the endowment or foundation.

To determine the optimal real estate allocation we begin with a portfolio that holds no real estate (0% allocation) and invests only in stocks (60%) and bonds (40%). We then gradually increase the real estate holding up to a maximum of 20%, with remaining funds invested in stocks/bonds 60%/40%. As we increase the weight of real estate, we record the percentage of the 5,000 iterations from the Monte Carlo simulation in which all the funds from the investment account are depleted, as well as the percentage of iterations that fail to achieve preservation of the original funds in real terms. We repeat this exercise with private real estate and with REITs as the real estate holding separately. The optimal allocations for the two predefined investor types are the real estate allocations that are associated with (1) the lowest percentage of fund depletion, and (2) the lowest percentage of failure to achieve funds preservation.

Finally, we examine the optimal allocation to private real estate and to REITs within a combined real estate allocation. To do so, we first fix the overall real estate allocation[5] and then vary the allocation of private real estate from 0% to 100% within the real estate holding while the remainder is invested in REITs. The private/public real estate allocations that are associated are associated with (1) the lowest percentage of fund depletion, and (2) the lowest percentage of failure to achieve funds preservation.

III. Results

A. Minimizing Shortfall Risk

In this section we examine the results of our simulations when minimizing shortfall risk, the probability that a spending rule is not sustainable over the full investment horizon (i.e. the balance in the investment account goes to zero before the end of the horizon). Table 2 shows the results of optimizing the allocation to private real estate for various horizons and withdrawal rates, and clearly shows that private market real estate can have a role in a portfolio designed to minimize shortfall risk. In many of the cases examined the optimal real estate allocation is 20%, the maximum allowed in the constrained simulations. Even the lowest allocations in the table, at more than 3% of the overall portfolio, are significant in an economic sense and, if such an allocation were to be adopted generally by the DC industry within their post-retirement glide paths, would represent a major change and a major source of capital flowing into private market real estate. Note that the optimal allocation to private real estate decreases for higher withdrawals rates; this is because the net of fees return to private real estate is low relative to the other asset classes and not able to keep up with the higher real withdrawal rates. This leads one to the conclusion that those individuals needing to spend a higher percentage of their retirement savings each period (presumably those with relatively little total savings relative to pre-retirement income) should allocate less to real estate than those individuals who are able to spend at a lower rate, relative to their wealth, in retirement.

Figure 1 presents the probabilities of shortfall for 25, 30 and 35 year investment horizons as the allocation to private real estate varies from 0% to 20%. For space reasons we present only the figure for a 4% withdrawal rate; results for other withdrawal rates show patterns that are qualitatively similar and are available from the authors on request. As should be expected, the longer the horizon the higher the shortfall risk, regardless of portfolio allocation. For our purposes the most interesting insight from Figure

[5] We fix the total real estate allocation to 20% and to 5% as two separate scenarios.

1 is that the shortfall risk varies very little across different private real estate allocations. For instance, Table 2 revealed the portfolio that minimized shortfall risk for a 30 year horizon and 4% withdrawal rate included a 20% allocation to private real estate; Figure 1 reaffirms this, but also reveals that the probability of shortfall at that allocation is 0.94% while the probability of shortfall is only 1.16% even with a zero allocation to private real estate. Thus, while the optimal portfolios do indeed include significant allocations to private real estate, the relatively small difference that such allocations would seem to make to minimizing shortfall risk leads one to doubt whether there is sufficient marginal value to attract investment from the DC market into private real estate, given the various institutional issues involved. However, it is important to note that our simulations assume that minimizing shortfall risk is the only goal of the investor; no allowance is made for other characteristics which may be of interest to some investors, such as increasing upside potential, minimizing tail risk, minimizing within-horizon volatility, etc.

Table 3 and Figure 2 present the results when considering an allocation to REITs rather than private real estate. When using public markets for the real estate allocation, in all cases the optimal allocation to real estate is at or close to the maximum 20% constraint, as shown in Table 3. These optimal REIT allocations are generally higher than those when private real estate is considered (except, of course, in those cases when both forms of real estate have optimal allocations at the 20% maximum). Figure 2 reveals even larger differences between private and public real estate allocations in minimizing shortfall risk. Unlike the results for private real estate, adding REITs to a portfolio generally decreases the shortfall risk. For example, for a 35 year horizon and a 4% withdrawal rate the probability of shortfall is 2.27% with no allocation to REITs, but falls to less than half of that (1.06%) at a 19% allocation to REITs. Especially for long horizon investors (presumably those retiring early or with longer expected life spans) an allocation to REITs can make a significant difference in the sustainability of the retirement spending plan.

To facilitate a comparison of the results on private real estate and REITs individually, Figures 3A and 3B plot the differences in shortfall probabilities when using one form of real estate but not the other in the portfolio. For instance, we first calculate the shortfall probability when the portfolio is invested with an x% allocation to private real estate and the remainder in a 60/40 stock/bond portfolio (e.g. the results from Figure 1) and then subtract the shortfall probability if the portfolio was invested with the same x% allocation to REITs and the remainder in a 60/40 stock/bond portfolio. We plot these differences across various allocation percentages and horizons, for withdrawal rate of 4% (Figure 3A) and 5% (Figure 3B). This allows a more direct comparison of public and private real estate in an “either/or” setting.

Looking at Figure 3A reveals that for almost all allocations to real estate, shortfall risk would be lower if REITs were used to fill the allocation. However, for small allocations, the difference in shortfall risk between private and public real estate is small, and in fact favors private in some cases. For instance, at allocations of 5% or less to real estate for a 25 year investment horizon, the use of private real estate would actually produce slightly lower shortfall risk. This is potentially important in practice, as in many cases alternative asset classes (especially private market assets) such as real estate are constrained to a fairly small allocation in portfolios. In a case where a portfolio is constrained to a small allocation to real estate, the results show little difference between using public or private real estate to fill the allocation, with a slight advantage to private. Conversely, if the portfolio is unconstrained (at least up to the 20% maximum we use in our simulations), REITs obviously outperform in terms of reducing shortfall risk.

Up to now, the results presented have dealt with cases where a portfolio was allocated to either private real estate or public real estate. A more realistic case, of course, would allow for the possibility of both, and it to this that we now turn. This is important not only because it relaxes the arbitrary constraint of choosing only one form of real estate, but also because it is closely related to the choices made by actual DC-eligible private real estate funds, which, as noted in the introduction, are often actually a combination of private real estate along with a REIT allocation for liquidity. To explore this issue we determine the combination of private real estate and REITs that would minimize shortfall risk. In implementing this in our Monte Carlo simulations, we first set an overall allocation to real estate in the portfolio, with the remainder invested in the 60/40 stock/bond portfolio. We then use the simulations to determine what combination of private and public real estate, adding up to the preset overall real estate allocation, would result in the lowest probability of financial ruin before the investment horizon. We test both a 20% overall allocation to real estate (as this was the maximum allocation used in our previous tests, and a common result for optimal allocation in both the private and public cases), as well as a 5% overall real estate allocation (to represent what would be a reasonably common allocation constraint to an alternative asset class in a portfolio). The results, showing the optimal combination of public and private real estate, are presented in Tables 4A and 4B.

Table 4A shows the real estate combinations minimizing shortfall risk when the overall real estate allocation is 20%. In this case, REITs are obviously dominant. Private market real estate has an optimal allocation of zero in most cases, and when it is positive it is very small. The positive allocations to private

market are small enough, and infrequent enough, that they could simply be due to random variation in the Monte Carlo outcomes. Consistent with the results of Figures 3A and 3B, which looked at each form of real estate in isolation, when the case where real estate forms a relatively high allocation in the overall portfolio REITs provide the best shortfall minimizing attributes.

Table 4B, in which the total allocation to real estate is constrained to 5%, tells a different story. In this case the optimal combination is much more evenly split between private and public market real estate, at least for the longer investment horizons of 30 and 35 years. While the optimal balance tilts more towards REITs as the withdrawal rate increases, in most cases there are significant allocations to both forms of real estate. For example, going back to our original consideration of the 4% Rule, based on the most common investment horizon in retirement planning research (30 years) the shortfall minimizing allocation to real estate would include 2.9% in private market real estate and 2.1% in REITs. It appears that, in at least some cases, both forms of real estate have a role to play in a portfolio designed to create sustainable spending patterns in retirement.

B. Maintaining Wealth

We now turn to the results under a different objective, that of spending a fixed, real percentage of initial wealth each year with a goal of maintaining a portfolio balance at the end of the investment horizon equal to inflation-adjusted initial wealth. As noted in the introduction, this can be thought of in two ways: (1) a retiree wishing to maintain a spending rule throughout their lifetime, while also leaving a bequest, or (2) a foundation or endowment concerned with income and wealth preservation. Foundation and endowments represent institutional investors with a goal of providing funding for an underlying cause or organization. Generally speaking, in most cases these investors want to maintain the spending power of their funding each year over time, and, since these investors are usually infinitely lived, they have a goal of (at least) maintaining the value of the portfolio over time. Hence our interpretation of them in a spending rule context with a goal of maintaining the value of the portfolio in real terms seems appropriate. [6] Of course, while foundations and endowments have infinite lives, we cannot run an infinite Monte Carlo

[6] We admit that a fixed, real spending rate is a gross oversimplification of the spending rules used by endowments and foundations in practice. As an example, the Yale endowment uses a spending rule each year based on a combination of the prior year's spending and a long term spending rate applied to current market value of the portfolio (see Swenson (2009), page 29). Spending rates at other endowments vary widely in amount and the actual rules used. Charitable foundations are legally required to spend at least 5% of their portfolio value each year

simulation. We again use a finite horizon simulation because (1) it is possible to implement, (2) doing so allows us to also retain the retirement with bequest interpretation, and (3) we believe that the goal of having, at the end of the horizon, the same portfolio value in real terms as at the beginning of the investment period encapsulates the infinite investment horizon of an endowment or foundation. Still, to better match the very long horizon nature of endowments and foundations we use longer investment horizons in this section for our Monte Carle runs, of 35, 40, 45, and 50 years.

Table 5 presents the results of our simulations when allocating to only private real estate along with stocks and bonds, with a goal of minimizing the risk of ending with wealth falling below inflation-adjusted initial levels. Private real estate has a significant allocation in the optimal portfolio for the lower withdrawal rates of 3.5% and 4%, with the optimal real estate allocation generally declining for longer investment horizons. Note that these allocations, when considering only private real estate and not public, are lower than those seen in Table 2 when the goal was only to maintain spending with no goal to maintain portfolio value. Further, Figure 4 shows that the probability of not maintaining inflation-adjusted wealth generally increases with the allocation to private real estate (only a 4% withdrawal rate is presented for space reasons, the results for other rates follow qualitatively similar patterns). The overall conclusion is that private real estate performs worse within a portfolio when the goal is to maintain spending, and to maintain the real value of the portfolio as well.

REITs, on the other hand, continue to play a strong role in an optimal portfolio when the goal includes maintenance of portfolio value over time. Table 6 shows that optimal allocations to REITs are near or at the self-imposed 20% maximum in all cases considered. Further, Figure 5 indicates that the probability of the ending inflation-adjusted portfolio value falling below the initial value decreases sharply as the allocation to REITs increases.

Finally, Tables 7A and 7B present the results when looking for an optimal combination of private and public real estate when the fixed overall allocation to real estate is fixed. As before, we use 20% and 5% overall allocations. As was the case when examining shortfall risk, REITs dominate when the real estate allocation is 20%. In all cases REITs account for all, or nearly all, of the real estate allocation when

to maintain their charitable status, but of course may spend a higher percentage. Our simplified spending rule is not meant to reflect the actual spending rules used by foundations and endowments exactly, but rather to be a general reflection of their desire to spend consistently over time and maintain value.

minimizing the probability of falling below initial wealth. When considering a 5% total allocation to real estate, as shown in Table 7B, the results are again split between REITs and private real estate. However, the weight is definitely more towards REITs than was the case for minimizing shortfall risk alone.

Overall, this section shows that real estate in the form of REITS can play a very significant role in portfolios with a goal of sustaining a spending pattern while also maintaining portfolio value. Private market real estate, however, is less attractive as an asset class when the goal includes maintenance of real portfolio value over time, as compared to the case when spending over a finite horizon is the only goal. In large part, this is likely due to the relatively low average net returns to private real estate seen over the 1978-2013 period and used in our simulations.

IV. Future Work

As we build on these results going forward, we intend on addressing several interesting issues. These include:

- To what extent do the larger fees charged on private market real estate affect results?
- What are the other characteristics of real estate (both private and public) that may affect their desirability in some spending rule based portfolios, such as tail risk, upside potential, within investment horizon volatility, ongoing income, etc.
- Why does private real estate seem to have a greater role when the overall real estate allocation is constrained to be relatively small share?

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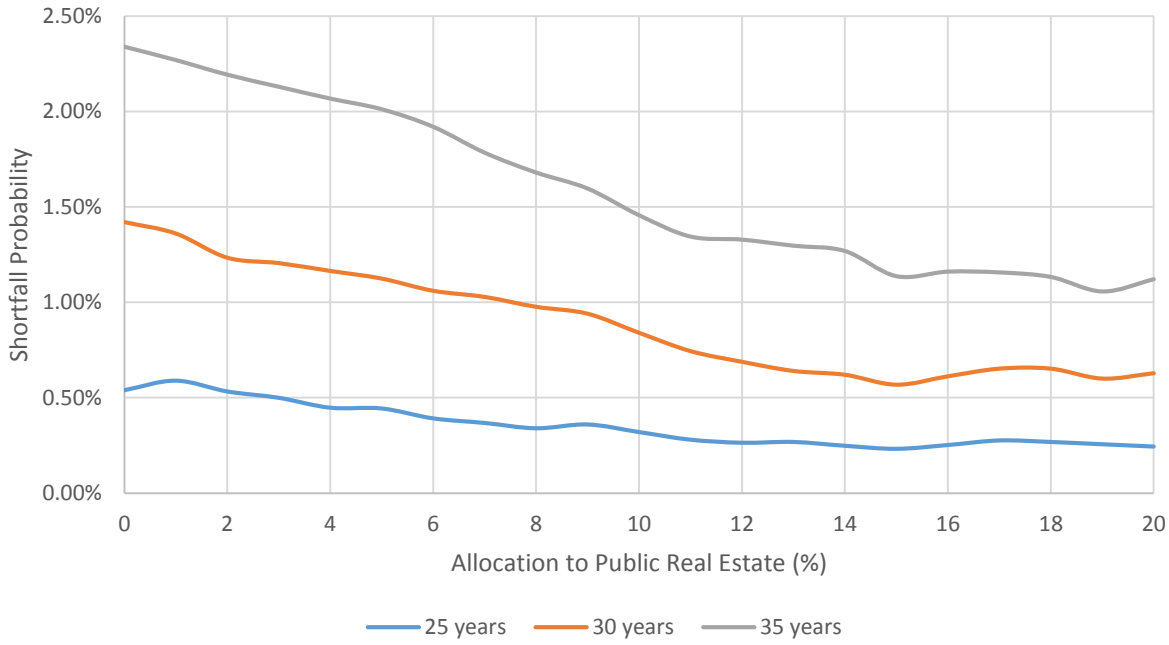
Figure 1: Shortfall Probabilities at 4% Withdrawal Rate with Private Real Estate Allocation

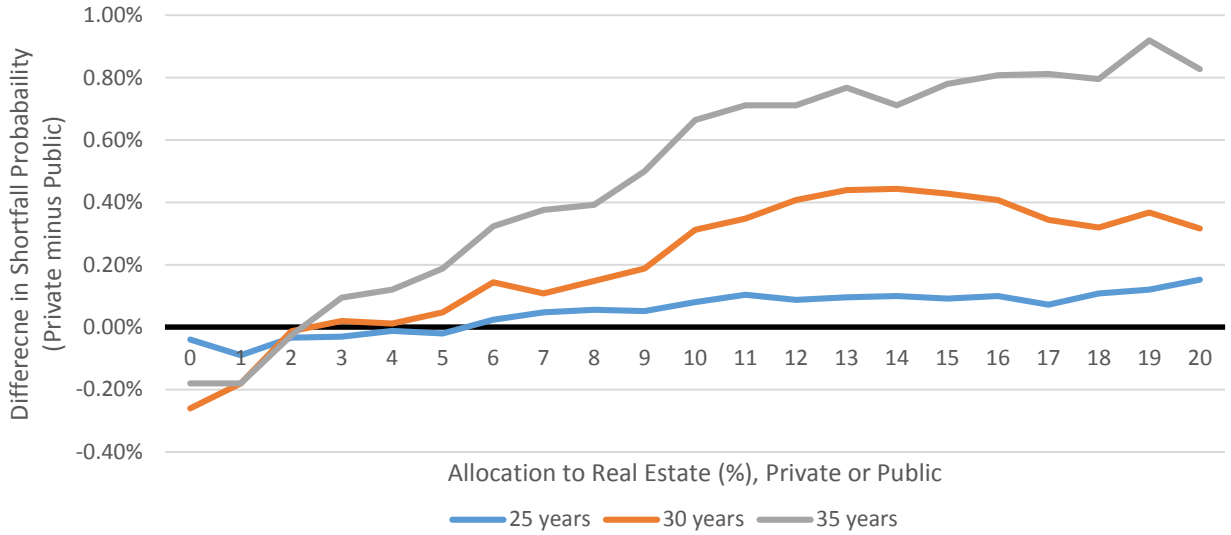
Notes: The allocation to real estate is limited to a 20% maximum. The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Figure 2: Shortfall Probabilities at 4% Withdrawal Rate with REIT Allocation



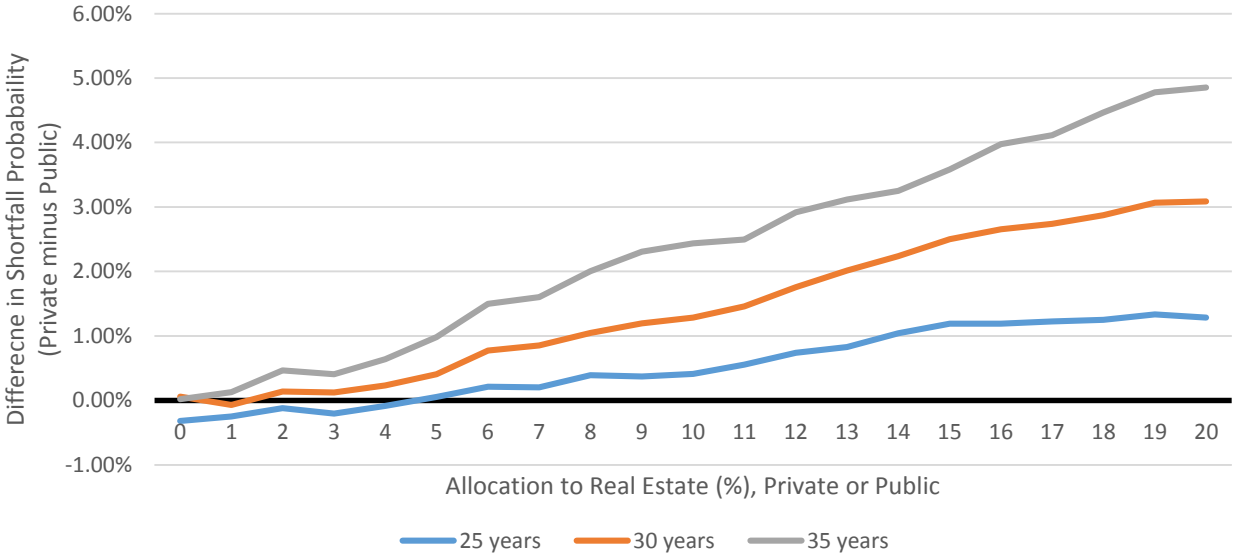
Notes: The allocation to real estate is limited to a 20% maximum. The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Figure 3A: Differences in Shortfall Probabilities When Allocating to Private or Public Real Estate - 4% withdrawal rate



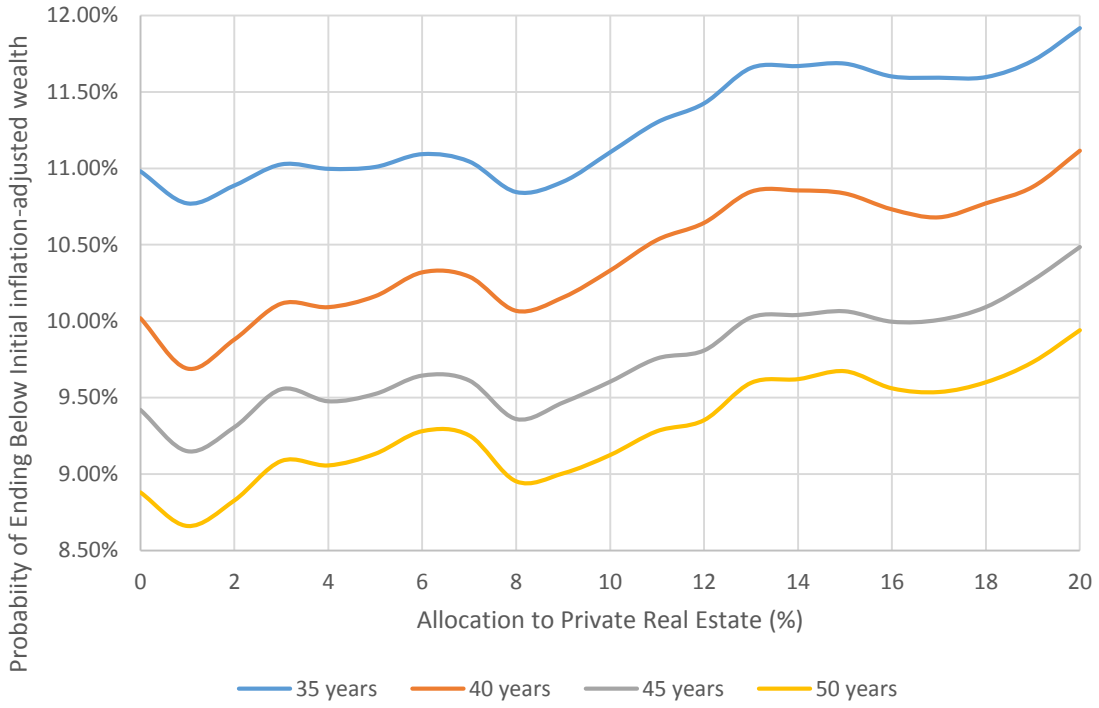
Notes: The figure represents the differences between the results of Figures 1 and 2.
For each allocation percentage the figure gives the differences between the shortfall probability for a portfolio including private real estate at that allocation and shortfall probability for a portfolio including public real estate at that same allocation. The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds.
Withdrawal rate is a percentage of initial wealth, in real terms.

Figure 3B: Differences in Shortfall Probabilities When Allocating to Private or Public Real Estate - 5% withdrawal rate



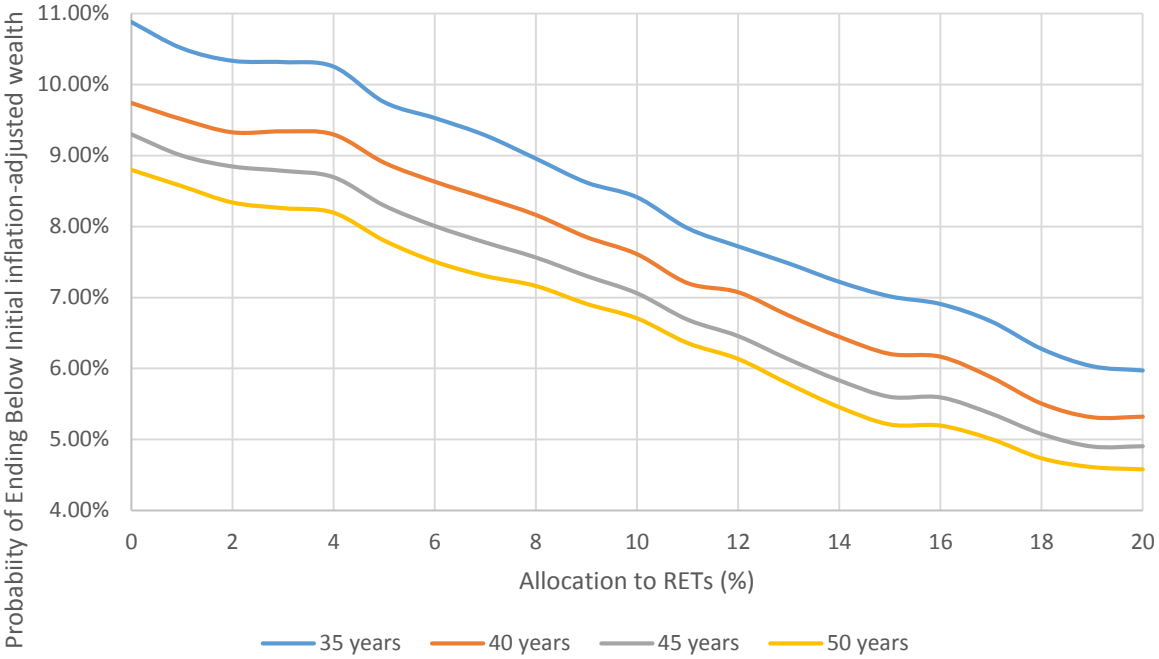
Notes: The figure represents the differences betwen the results of Figures 1 and 2.
For each allocation percentage the figure gives the differences between the shortfall probability for a portfolio including private real estate at that allocation and shortfall probability for a porotlfio including public real estate at that same allocation. The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds.
Withdrawal rate is a percentage of initial wealth, in real terms.

Figure 4: Probabilities of Ending Below Initial Inflation-Adjusted Wealth at 4% Withdrawal Rate with Private Real Estate Allocation



Notes: The allocation to real estate is limited to a 20% maximum. The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Figure 5: Probabilities of Ending Below Initial Inflation-Adjusted Wealth at 4% Withdrawal Rate with REIT Allocation



Notes: The allocation to real estate is limited to a 20% maximum. The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Table 1: Annual Returns, 1978 – 2013

	Bonds	Stocks	REITs	Private Real Estate	Inflation
Mean	0.081	0.134	0.140	0.079	0.038
Std. Deviation	0.070	0.172	0.173	0.098	0.029

Table 2: Allocation to Private Real Estate Minimizing Probability of Shortfall

Withdrawal Rate	Horizon (years)	Private Real Estate Allocation
3.50%	25	20.00%
4.00%	25	19.65%
4.50%	25	7.91%
5.00%	25	5.12%
3.50%	30	20.00%
4.00%	30	20.00%
4.50%	30	10.00%
5.00%	30	8.44%
3.50%	35	20.00%
4.00%	35	14.08%
4.50%	35	9.51%
5.00%	35	3.33%

Notes: The allocation to real estate is limited to a 20% maximum. The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Table 3: Allocation to REITs Minimizing Probability of Shortfall

Withdrawal Rate	Horizon (years)	Public Real Estate Allocation
3.50%	25	20.00%
4.00%	25	20.00%
4.50%	25	20.00%
5.00%	25	19.36%
3.50%	30	20.00%
4.00%	30	19.72%
4.50%	30	20.00%
5.00%	30	19.21%
3.50%	35	20.00%
4.00%	35	19.57%
4.50%	35	20.00%
5.00%	35	18.88%

Notes: The allocation to real estate is limited to a 20% maximum. The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Table 4A: Allocations to Private and Public Real Estate Minimizing Probability of Shortfall, 20% total allocation to Real Estate

Withdrawal Rate	Horizon (years)	Private Real Estate Allocation	REIT Allocation
3.50%	25	0.00%	20.00%
4.00%	25	0.00%	20.00%
4.50%	25	2.16%	17.84%
5.00%	25	2.49%	17.51%
3.50%	30	0.00%	20.00%
4.00%	30	1.27%	18.73%
4.50%	30	0.00%	20.00%
5.00%	30	0.00%	20.00%
3.50%	35	0.00%	20.00%
4.00%	35	4.25%	15.75%
4.50%	35	0.00%	20.00%
5.00%	35	0.00%	20.00%

Notes: The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Table 4B: Allocations to Private and Public Real Estate Minimizing Probability of Shortfall, 5% total allocation to Real Estate

Withdrawal Rate	Horizon (years)	Private Real Estate Allocation	REIT Allocation
3.50%	25	0.00%	5.00%
4.00%	25	0.00%	5.00%
4.50%	25	0.00%	5.00%
5.00%	25	1.55%	3.45%
3.50%	30	2.51%	2.49%
4.00%	30	2.90%	2.10%
4.50%	30	0.00%	5.00%
5.00%	30	0.73%	4.27%
3.50%	35	2.67%	2.33%
4.00%	35	1.65%	3.35%
4.50%	35	1.47%	3.53%
5.00%	35	0.88%	4.12%

Notes: The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Table 5: Allocation to Private Real Estate Minimizing Probability of Ending Below Inflation-Adjusted Initial Wealth

Withdrawal Rate	Horizon (years)	Private Real Estate Allocation
3.50%	35	14.65%
4.00%	35	7.56%
4.50%	35	1.75%
5.00%	35	0.25%
3.50%	40	14.89%
4.00%	40	7.57%
4.50%	40	2.00%
5.00%	40	0.14%
3.50%	45	8.52%
4.00%	45	7.64%
4.50%	45	1.00%
5.00%	45	0.00%
3.50%	50	8.88%
4.00%	50	7.79%
4.50%	50	1.06%
5.00%	50	0.00%

Notes: The allocation to real estate is limited to a 20% maximum. The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Table 6: Allocation to REITs Minimizing Probability of Ending Below Inflation-Adjusted Initial Wealth

Withdrawal Rate	Horizon (years)	Public Real Estate Allocation
3.50%	35	20.00%
4.00%	35	18.53%
4.50%	35	20.00%
5.00%	35	19.52%
3.50%	40	20.00%
4.00%	40	18.77%
4.50%	40	20.00%
5.00%	40	19.49%
3.50%	45	19.69%
4.00%	45	18.09%
4.50%	45	20.00%
5.00%	45	19.48%
3.50%	50	19.20%
4.00%	50	18.11%
4.50%	50	20.00%
5.00%	50	19.33%

Notes: The allocation to real estate is limited to a 20% maximum. The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Table 7A: Allocations to Private and Public Real Estate Minimizing Probability of Ending Below Initial Inflation-Adjusted Wealth, 20% total allocation to Real Estate

Withdrawal Rate	Horizon (years)	Private Real Estate Allocation	REIT Allocation
3.50%	35	0.00%	20.00%
4.00%	35	1.34%	18.66%
4.50%	35	0.00%	20.00%
5.00%	35	0.00%	20.00%
3.50%	40	0.00%	20.00%
4.00%	40	0.00%	20.00%
4.50%	40	0.00%	20.00%
5.00%	40	0.00%	20.00%
3.50%	45	0.00%	20.00%
4.00%	45	0.06%	19.94%
4.50%	45	0.00%	20.00%
5.00%	45	0.00%	20.00%
3.50%	50	0.00%	20.00%
4.00%	50	1.48%	18.52%
4.50%	50	0.00%	20.00%
5.00%	50	0.00%	20.00%

Notes: The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.

Table 7B: Allocations to Private and Public Real Estate Minimizing Probability of Ending Below Initial Inflation-Adjusted Wealth, 5% total allocation to Real Estate

Withdrawal Rate	Horizon (years)	Private Real Estate Allocation	REIT Allocation
3.50%	35	1.11%	3.89%
4.00%	35	0.63%	4.37%
4.50%	35	1.13%	3.87%
5.00%	35	0.39%	4.61%
3.50%	40	1.17%	3.83%
4.00%	40	0.00%	5.00%
4.50%	40	1.33%	3.67%
5.00%	40	0.25%	4.75%
3.50%	45	1.39%	3.61%
4.00%	45	0.99%	4.01%
4.50%	45	1.24%	3.76%
5.00%	45	0.44%	4.56%
3.50%	50	1.41%	3.59%
4.00%	50	1.06%	3.94%
4.50%	50	1.05%	3.95%
5.00%	50	0.18%	4.82%

Notes: The portion of the portfolio not in real estate is allocated on a 60/40 basis to stocks/bonds. Withdrawal rate is a percentage of initial wealth, in real terms.